Volatility Cluster and Herding

Friedrich Wagner

Institut für Theoretische Physik, Universität Kiel, Leibnizstrasse 15, D-24098 Kiel, Germany

Abstract

Stock markets can be characterized by fat tails in the volatility distribution, clustering of volatilities and slow decay of their time correlations. For an explanation models with several mechanisms and consequently many parameters as the Lux-Marchesi model have been used. We show that a simple herding model with only four parameters leads to a quantitative description of the data. As a new type of data we describe the volatility cluster by the waiting time distribution, which can be used successfully to distinguish between different models.

1 Introduction

Financial stock markets exhibit several universal properties, socalled 'stylized facts'[1,2]. There are no time correlations in the index for time scales larger than a few minutes [3,4]. The distribution of relative changes or returns are power law distributed at large values with an exponent in the order of 3-4 [4,5]. The absolute returns or volatilities exhibit multifractal moments [6]. In the time development of the volatilities we find two features. The correlation decays very slowly. Wether this decay is described by a power law with a small exponent or an exponential cannot be decided. A power law would be equivalent with a $1/f$ noise [4]. The Joseph effect [7] manifests itself by volatility clusters, that large changes of the index occur preferably at neighbouring times.

These facts are interesting enough to motivate also physicists to look for microscopic models for an at least partial explanation. By microscopic models we mean a market consisting of agents following certain trading strategies together with a mechanism for the evolution of the price or the index. Many different approaches have been persued in the recent years (for reviews see [8] or recently [9]). All these models use inhomogenous agents in contrast to the effective market hypothesis [3]. The agents are distinguished by differences in the behaviour as noise traders or fundamentalists [10–13], the wealth [14],

the bidding prices [15], the trading thresholds [16] or the length of memory [17]. Alternatively equal agents select different strategies depending on an utility function [18–21]. The mechanism for creating volatility clusters may be a memory, nonlinear coupling between the price and agent parameters or the herding effect. The latter may be acchieved by a next neighbour interaction as in statistical mechanics [16,19–21] or explicitely in the dynamics [11,12]. A model including all three mechanisms is the Lux-Marchesi model [12] (hereafter abbreviated by LMM). If such a model accounts successfully for the stylized facts as it happens for LMM one would like to know whether all three mechanisms are needed or one alone is already sufficient. In the present paper we investigate the question whether herding alone can explain the stylized facts.

As a model we use a simplified version of the LMM. As pointed out in [11] there must be three type of agents for which we take noise traders (pessimists and optimists) and fundamentalists as in LMM. An important unique feature of the LMM is that agents can change their oppinion. Herding is described by transition rates proportional to the number of agents in the new state. If we neglect the dependence of the agent numbers on the price we can use the ant formalism of Kirman [22] or its generalization by Aoki [23]. Since the price has disappeared from the dynamic we have to find a phenomenological relation in terms of the agent numbers. For that we use the observation [24], that in the LMM the dynamics of the price is much faster than that of the agents and therefore the equilibrium relation can be used. Assuming a constant number of fundamentalists the Kirman model can acchieve qualitative agreement with the data [24]. However, we will show that the time structure of the clusters is in disagreement with the data. Moreover there exists a winning strategy for the fundamentalists, since noise traders are not allowed to become fundamentalists. Therefore we use a three component herding model where herding is included also for the fundamentalists as in [11].

The paper is organized in the following way. In the next section we describe the data (German DAX index [25]). We use the volatility distribution, the the correlation function of the volatility and as a new feature the waiting times between volatilities larger than a certain minimum value. Section 3 is devoted to the description of the agent strategies and the price relation. For large numbers of agents the Kirman model can be written in terms of a Fokker-Planck equation. Instead of using transition probabilities we formulate the dynamics by the corresponding Langevin equation for the fraction of agents. This model and its consequences are described in section 4. The formalism can be easily generalized for three components (section 5). Our conclusions are contained in section 6.

2 German DAX index

The indices $p(t)$ of stock markets show a long term increase. This is a rather small effect, f.e. for the German DAX index 0.03%/day over 30 years. Apart from that there are no time correlations on time scales larger than a few minutes [3,4]. In contrast the absolute values of the return or the volatility defined by

$$v(t) = \frac{1}{p(t)} \left| \frac{\Delta p}{\Delta t} \right| \tag{1}$$

exhibits the stylized facts mentioned in section 1, if we consider dayly variations. Unfortunately also the values of $v(t)$ averaged over few years show long term variations, as pointed out in [26]. To avoid a long term bias we chose the dayly quoted values of the DAX in the time period 1996-1999 corresponding to 784 trading days, where the three month average of $v(t)$ is compatible with a constant value $v_{av} = 0.011$. In Fig.1 we show the distribution of v/v_{av} on a

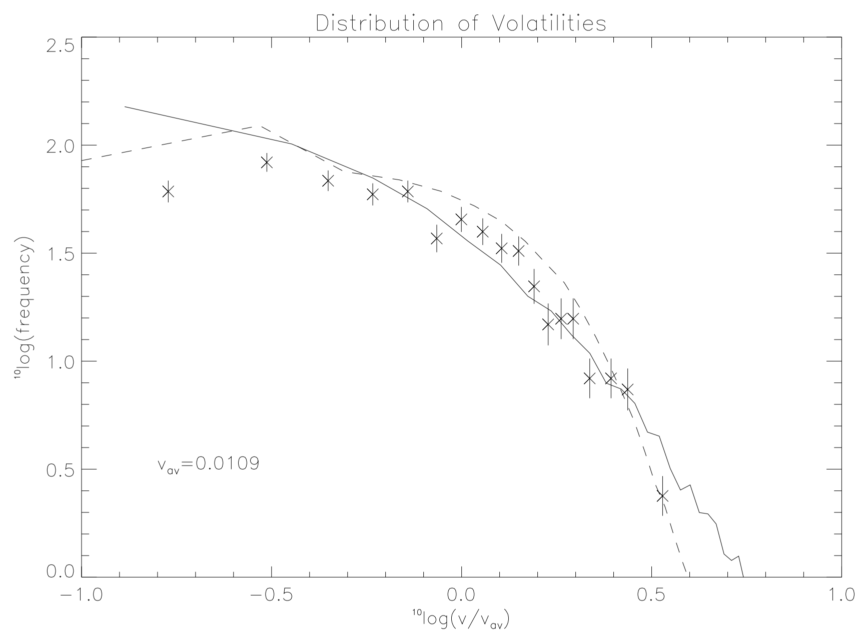

Fig. 1. The distribution of normalized volatilities for the DAX is shown on a double log scale (data points). The solid (dashed) line show the prediction of the three component model (Lux-Marchesi model).

double log scale for these data. The use of normalized volatilities [4] is important when comparing the data with models. Since v_{av} defines a time scale, the translation of computer time steps in a simulation into a real time is taken into account using the normalized volatilities v/v_{av}. As one sees from Fig.1

both LMM and the three component model to be discussed in section 5 are compatible with the data. It is difficult to decide whether there is a power law behaviour at large v or not. The errors of the data points shown in Fig.1 (and in the subsequent figures) are computed by binning the volatilities and taking $\sqrt{N_b}$ as statistical error, if the bin contains N_b events.
We quantify the effect of volatility clustering by the waiting times τ between occurences of volatilities larger than a minimum value v_{min} measuring the time $t = \tau \Delta t$ in units of $\Delta t = 1$ day. If the volatilities $v(\tau)$ are statistically independent the probability $p_w(\tau)$ for a waiting time $\tau = 1, 2, ...$ is given by

$$p_w(\tau) = \frac{1}{\tau_0 - 1} \left(\frac{\tau_0 - 1}{\tau_0} \right)^{\tau} \quad , \tag{2}$$

where the average waiting time τ_0 is the inverse of the cumulative probability $P(v \geq v_{min})$. In Fig.2 we show $p_w(\tau)$ for different v_{min}. For $v_{min} << v_{av}$ it

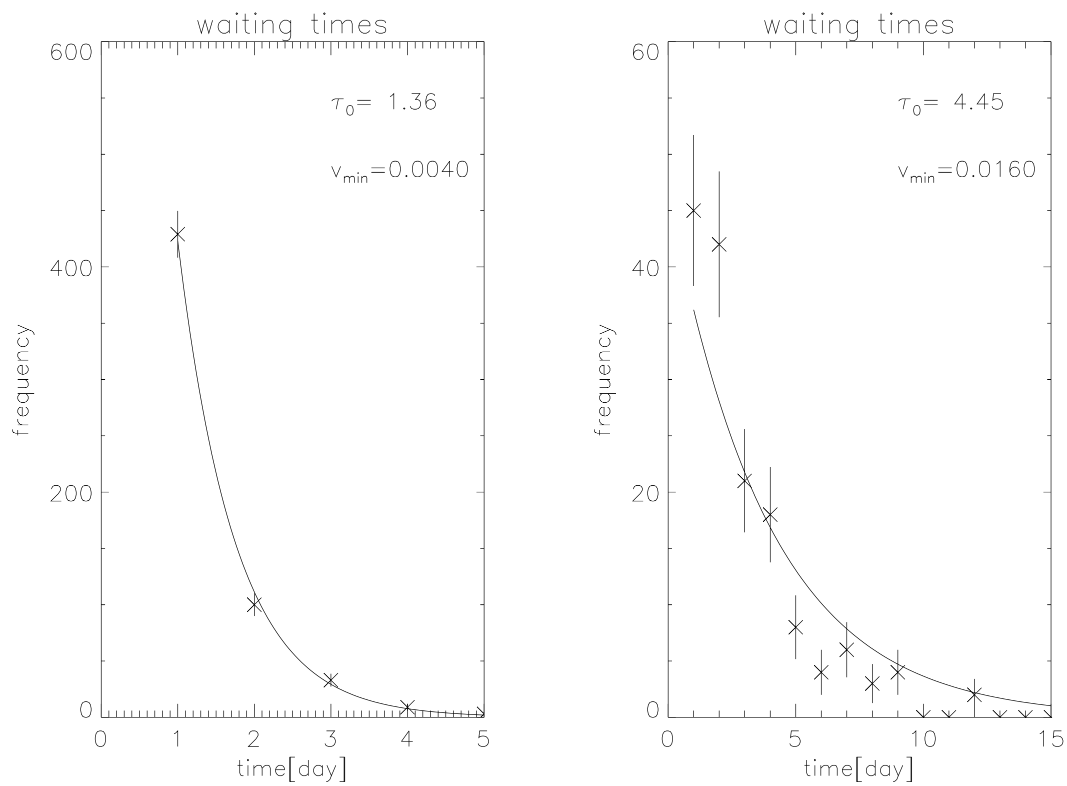

Fig. 2. The distribution of the waiting time for a volatility larger than v_{min} for $v_{min} = 0.004$ and 0.016. The curves correspond to independent volatilities and are given by equ.(2) using the observed average waiting time τ_0.

agrees with (2) indicating that small changes of the index can be considered as independent. For values $v_{min} \sim v_{av}$ we see an excess at small and large

value of τ and a depletion near $\tau \sim \tau_0$ as expected from formation of volatility clusters. The experimental distribution can be characterized by the average waiting time $\tau_0(v_{min})$ and a clustering parameter $\gamma(v)$ defined by

$$\gamma(v_{min}) = \sum_{\tau=1}^{[2\tau_0]} \mathrm{sign}(\tau_0 - \tau) \left(\frac{N(\tau)}{N_0} - p_w(\tau) \right) / \sum_{\tau=1}^{[\tau_0]} p_w(\tau) \quad , \tag{3}$$

where $N(\tau)$ corresponds to the observed number of events at time τ and N_0 to the total number. The clustering parameter γ is proportional to the difference of the areas between the data and the curve $p_w(\tau)$ expected from independent volatilities in the intervalls $\tau < \tau_0$ and $\tau_0 < \tau < 2\tau_0$. The average waiting time $\tau_0(v)$ is compared in Fig.3 with independent Gaussean distributed volatilities

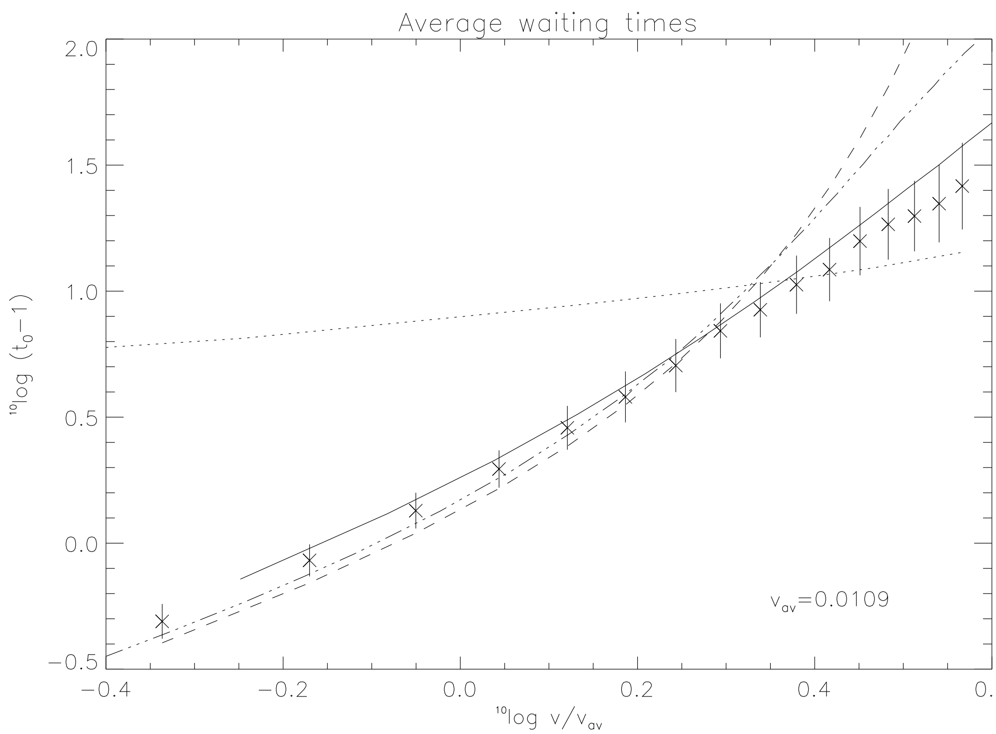

Fig. 3. The average waiting time $\tau_0(v)$ for volatilities larger than v as function of the normalized volatility on a double log scale. The solid (dotted,dashed,dotted-dashed) line show the prediction of the three component model (Kirman model, random walk, Lux-Marchesi model).

given by

$$\frac{1}{\tau_0} = \frac{1}{\sqrt{\pi}} \,\mathrm{erfc}\left(\frac{v}{v_{av}\sqrt{\pi}}\right) \tag{4}$$

and the LMM, which both are remarkably close to the data for $v < 2v_{av}$. Again we used the scaled values v/v_{av} of the minimum volatility $v = v_{min}$, which allows an absolute prediction for one parameter models as in equ. (4). In Fig.4 the cluster parameter $\gamma(v)$ from equation (3) is shown as function of

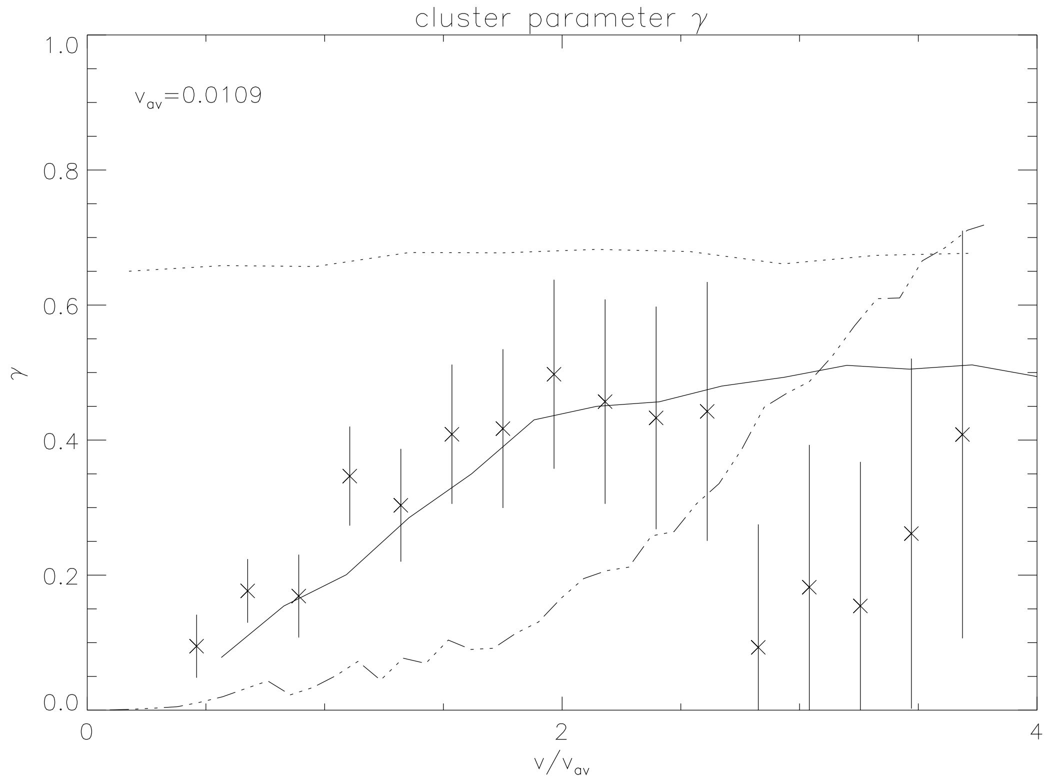

Fig. 4. The data show the cluster parameter γ defined by equ. (3) for the DAX as function of the normalized volatility. The solid (dotted, dotted-dashed) line show the prediction of the three component model (Kirman model, component model (Kirman model, Lux-Marchesi model). $\gamma = 0$ corresponds to no clustering.

v/v_{min}. The linear increase of $\gamma(v_{min})$ shows that there exists clustering on all scales of v. In the LMM and the Kirman model to be dicussed in section 4 the cluster are generated by an intermittency effect by two almost stable states. In the second model the fluctuations outside the cluster are almost zero and during the cluster of the same size leading to constant γ and τ_0 (see Fig.3). The reversed happens in the LMM. Here the fluctuations are independent for $v < v_{av}$ and the clustering begins only at large values of v. Note that the LMM has not been adjusted to these data. In view of the many parameters in this model the disagreement should not to be taken too seriously. As last experimental quantity we will use the correlation function defined by

$$C(\tau) = \frac{\sum_{t=1}^{T}(v(t)v(t+\tau) - v_{av}^2)}{\sum_{t=1}^{T}(v(t)v(t) - v_{av}^2)} \quad . \tag{5}$$

A meaningful experimental determination of $C(\tau)$ requires much more statistics than used in the previous figures. Therefore we use all DAX data from 1969 to 2002. The correlation function for the volatility is shown in Fig.5 on a log-linear scale. The data show a strong decrease at small values of τ which compares well with the prediction of the LMM. It fails to account for the values at large τ, which in turn are in agreement with the three component model.

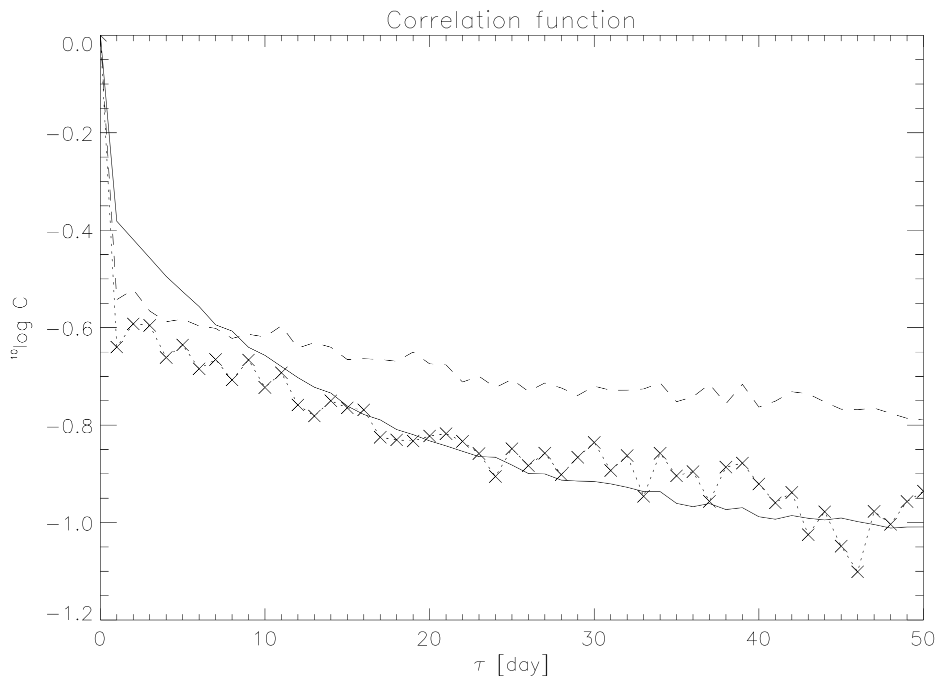

Fig. 5. The correlation function for the DAX (crosses) on a log-linear scale. An exponential decay would correspond to a straight line. The solid (dashed) line gives the prediction of the three component model (Lux-Marchesi model).

3 Equation of state for the price

In order to explain the features of the DAX discussed in the previous section 3 we introduce as in the LMM[12] N agents, which can adopt one of the following strategies. Optimistic (pessimistic) noise traders buy (sell) at each time one unit of stock. Fundamentalistic traders change their stock proportional to the difference of the actual price $p(t)$ and an assumed true value p_f. The system is characterized by the numbers $n_i(t)$ of agents following strategy $i = +, -f$ (optimistic, pessimistic and fundamentalistic) and the price $p(t)$. In general the dynamic will couple $n_i(t)$ and $p(t)$ in a complicated way. The new price

$p(t+\Delta t_p)$ will depend on the demand D and the supply S. To avoid dependence on splitting orders it should depend only on the difference $\omega = D - S$ which is given in our case by

$$\omega(p) = \frac{n_f}{p_0}(p_f - p) \; + \; n_+ - n_- \quad . \tag{6}$$

The following fairly general form of a price dynamic

$$p(t + \Delta t_p) = p(t) \; g\left(\frac{\lambda(t)}{\lambda(t) - \omega(p)}\right) \tag{7}$$

has been discussed by Helbing et al.[27]. In relation (7) $\lambda(t)$ can describe a reservoir of stocks to fulfill any order or the market liquidity. The montonically increasing function g must satisfy $g(1) = 1$. For the following it is irrelevant wether one assumes an exponential form [28], a linear function [12] or a power law[27]. Motivated by the simulation of the LMM we assume the price changes due to to the dynamic (7) occur at a much smaller time scale Δt_p than changes of $n_i(t)$ or $\lambda(t)$ at the scale Δt. Therefore the dynamic (7) will reach a fixed point before any changes in $n_i(t)$ or $\lambda(t)$ occur. The mapping (7) has the stable fixed point $\omega(p) = 0$ if the stability condition

$$\left|1 - g'(1)\frac{p}{p_0}\frac{n_f}{\lambda}\right| \; < \; 1 \tag{8}$$

holds, which can be achieved by sufficient large liquidity λ. From the fixed point condition $\omega(p) = 0$ we obtain the equation of state for the price

$$p = p_f + p_0 \, \frac{n_+ - n_-}{n_f} \quad . \tag{9}$$

This relation has been also proposed by Alfarano and Lux [24]. Alternatively one can consider (9) as a phenomenological relation expressing that high (low) price are expected, if the optimists (pessimists) and a price p_f if fundamentalists dominate the market. In order to avoid negative prices the second term in relation (9) must be much smaller than p_f. Since the average returns for the DAX or other markets are in the order of 1% the probability for a negative price is very small. Neglecting quadratic terms in v we find for the volatility (1)

$$v(t) = \frac{p_0}{p_f n_f} \left|\frac{\Delta n_+}{\Delta t} - \frac{\Delta n_-}{\Delta t}\right| \quad . \tag{10}$$

4 Kirman model

In this section we simplify the model even further by keeping the number of fundamentalists constant. This serves mainly to explain the method. It can be easily generalized to three components (see section 5). It has been introduced by Kirman [22] for volatility clustering and behaviour of ants. The noise traders follow the strategies denoted by $i = +, -$. The transition rate π per unit time for one agent to switch from strategy i to k is given by

$$\pi(i \to k) = a_{ik} + b_{ik} \cdot n_k \quad . \tag{11}$$

The number of noise traders $n_c = n_+ + n_-$ is to be kept constant. The constants a describe a spontaneous rate and b correspond to the herding behaviour. For symmetry between optimistic and pessimistic traders a and b do not depend on i, k. The property $b_{ik} = b_{ki}$ has the remarkable consequence [22,29] that the equilibrium distribution $w_0(n_+)$ exhibits non thermodynamical behaviour in the sense that it becomes for large n_c a non trivial function of

$$x = \frac{n_+ - n_-}{n_+ + n_-} \quad , \tag{12}$$

instead a Gaussean around the mean value of x with a width $\propto 1/\sqrt{n_c}$. This solves a problem in the LMM and other models which become trivial in the limit of many agents [30]. The transition probabilities (11) lead to a master equation. In the limit of large n_c it can be written in the form of a Fokker-Planck equation for the distribution $w(x,t)$:

$$\frac{\Delta w(x,t)}{\Delta t} = \frac{\partial}{\partial x}\left[-A_x(x)\, w(x,t) + \frac{1}{2}\frac{\partial}{\partial x} D_x(x)\, w(x,t)) \right] \tag{13}$$

with a drift term

$$A_x(x) = -2ax \tag{14}$$

and a diffusion term

$$D_x(x) = 2b(1 - x^2) \quad . \tag{15}$$

The equilibrium distribution $w_0(x)$ depends only on the ratio $\epsilon = a/b$ of the rate constants

$$w_0(x) = \frac{(1 - x^2)^{\epsilon - 1}}{B(\epsilon, 1/2)} \tag{16}$$

with the normalization given by Euler's B-function. If a system is described by a Fokker-Planck equation, we can replace the original dynamic (11) for the individual agents by a Langevin equation for x

$$\frac{\Delta x}{\Delta t} = A_x + \sqrt{\frac{D_x}{\Delta t}}\,\eta_t \tag{17}$$

with a white noise η_t. In equations (13) and (17) no dependence on n_c appears. It is hidden, however, by the condition, that the time scale Δt and the rates must satisfy

$$a\Delta t,\ b\Delta t\ \propto\ 1/n_c << 1 \quad , \tag{18}$$

if the dynamic (17) should represent the original dynamic (11). In the numerical simulation we use (17) in the form

$$x(t+1) = x(t) - \epsilon\rho x + \sqrt{\rho(1-x^2)}\,\eta_t \tag{19}$$

with an effective rate $\rho = 2b\Delta t$. Using relation (10) for the volatility, we arrive at a model similar to the one proposed by Alfarano and Lux [24]

$$v(t) = v_0\,|x(t+1) - x(t)| \quad . \tag{20}$$

Our version differs from [24] that in the latter the dynamic (11) is used and the drift term is replaced by a reflecting boundary at $x^2 = 1$ as in the LMM. For small x the system is described in both cases by a random walk. Near $|x| = 1$ the diffusion vanishes and the system remains a long time T_w near the boundary. The average of the return time T_w can be computed analytically [31]. For small ϵ we obtain

$$T_w = \frac{1}{b\epsilon}\ +\ \frac{1}{b}\ln\frac{e}{2}\ +\ o(\epsilon) \quad . \tag{21}$$

Leaving after T_w the boundary $x^2 = 1$, volatility clusters are formed by the diffusion mechanism (19) with a time scale $1/b << T_w$. In Fig.6 the results of a simulation are shown with $\epsilon = 0.02$ and $\rho = 0.02$. The left part gives $x(t)$ for $4 \cdot 10^4$ time steps. The accumulation of points near $|x| = 1$ is clearly seen. In the middle part we show the distribution of the volatility as function of v/v_{av}. The behaviour at large v is compatible with the data, but not the peak at small v. On the right we show the correlation function of the volatility on log-linear scale. It agrees with the observed correlation (see Fig.5), that a sharp drop at small τ is followed by an exponential decay as indicated by the straight line in Fig.6. However, the drop is much to small and the decay too

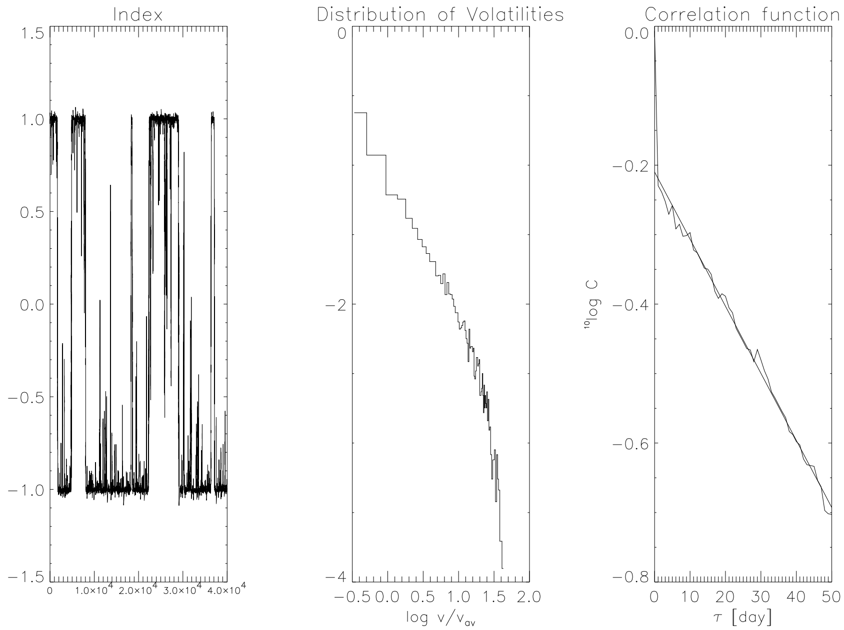

Fig. 6. Simulation of the Kirman model. The left part shows the time serie $x(t)$, the middle part the volatility distribution and the right part the correlation function given by equ. (5). The straight line corresponds to an exponential fit $\exp(-\gamma\tau)$ with $\gamma = 0.022$.

strong as compared to the data. One could get agreement with ϵ values near 0.5, which would give totally unrealistic volatility distributions. Apart from the peak at small volatilities and the quantitatively wrong correlations the Kirman model exhibits further unwanted features. Since the diffusion is fast the value of the volatilities in a cluster are always of the same size v_0 given by (20) and do not reproduce small size clustering seen in Fig.3 and Fig.4. The index values are bounded by

$$|p - p_f| \leq \frac{p_0}{n_f} \quad . \tag{22}$$

Such a bound apart being unrealistic leads to a winning strategy for the fundamentalists in contradiction the efficient market hypothesis [3]. These deficiencies can be cured by using a three component model, where noise traders can change into fundamentalists.

5 Three component model

The three component model is obtained, if one includes in the agent dynamic (11) also the fundamentalists ($i, k = +, -, f$). The system is described by the

fraction x as in equ. (12) and the ratio of noise traders to fundamentalists

$$u = \frac{n_+ + n_-}{n_f} \quad . \tag{23}$$

The total number $N = n_+ + n_- + n_f$ is to be kept constant. In order to obtain a nontrivial distribution in x, u in the limit of large N we require the a symmetric herding matrix b_{ik}. The rate parameters should be symmetric between optimistic and pessimistic noise traders leading to

$$b_{+k} = b_{+k} \ , \quad a_{+k} = a_{-k} \ , \quad a_{k+} = a_{k-} \quad . \tag{24}$$

There remain only five independent rate constants in the matrices a and b. A last condition comes from an integrability condition for the existence of a master equation, as discussed by Aoki [23]. This reads in our notation

$$\frac{a_{+-}}{b_{+-}} = \frac{a_{f-}}{b_{f-}} \quad . \tag{25}$$

Now we perform the same steps from the transition rates to the Langevin equation for x and u as in the Kirman model. The equation for x is the same as equ. (19). For u we obtain a second equation

$$u(t+1) = u(t) + A_u(u) \ + \ \sqrt{D_u(u)}\,\eta_t \tag{26}$$

with a drift term

$$A_u(u) = (1+u)\,[(1-\epsilon_3/2)u + \epsilon]\,\rho_3 \tag{27}$$

and a diffusion term

$$D_u(u) = \rho_3\ u\ (1+u)^2 \ , \tag{28}$$

where we introduced in analogy to (19) the effective rate $\rho_3 = 2b_{+f}\Delta t$ and the ratio $\epsilon_3 = a_{+f}/b_{+f}$. The expression for the volatility is given by

$$v(t) = v_0\,|u(t+1)x(t+1) - u(t)x(t)| \quad . \tag{29}$$

The equilibrium distribution for the fractions x and u factorizes into a product

$$w_0(x,u) = \frac{(1-x^2)^{\epsilon-1}}{B(\epsilon,1/2)} \cdot \frac{u^{2\epsilon-1}(1+u)^{-(\epsilon_3+2\epsilon)}}{B(2\epsilon,\epsilon_3)} \quad . \tag{30}$$

In the two component model volatility clusters are due to a large return time for $\epsilon \to 0$. Unfortunately, small ϵ are responsible for the unwanted behaviour $\propto v^{-1}$ in the volatility distribution. In the three component model one can avoid this drawback by a large time for a change from a small value u_0 to a large value u_1 given by

$$T(u_0 \to u_1) \propto (1 + u_1)^{\epsilon_3 - 1} , \tag{31}$$

whereas the reversed time $T(u_1 \to u_0)$ is independent of u_1. For simulations of the model the parameters ϵ and ϵ_3 can be determined from the equilibrium distribution of v. Our choice $\epsilon = 2$ and $\epsilon_3 = 6$ give a reasonable description of volatility distribution as can be seen from Fig.1. With the rate parameters $\rho = 0.001$ and $\rho_3 = 0.01$ the empirical average waiting times shown in Fig.3 and the cluster parameter γ shown in Fig.4 can be reproduced. Especially the u dependent return time (31) leads to clustering also for small v, as seen in Fig.4. With these values the correlation function shown in Fig.5 can be predicted, which agrees with the data except in the transition region between the drop and the slow decay. Since we are using normalized volatilities, only the ratio $\rho/\rho_3 = 0.1$ is important for waiting times and volatility distributions. It means that changes in the index are mainly caused by transitions between noise traders and fundamentalists, and not by changes between optimists and pessimists. The ratio $\epsilon_3/\epsilon = 3$ implies that changes from noise traders to fundamentalists are more influenced by the spontaneous rate than the reversed process. Since the spontaneous rate may represent private information, this seems not to be unreasonable. From the average value of $< u >= 0.4$ we find that the fraction of noise traders is only roughly 30%.

6 Conclusions

The empirical data for a stock market as the DAX can be explained by a model based on herding alone. Especially the time structure of the volatility clustering can rule out a two component model. At least three components are needed. Since the description of the data is comparable or better than the predictions of the Lux-Marchesi model, one may conclude that herding is the dominant mechanism in the LMM, and the nonlinear coupling of the price evolution to the transition probabilities is less important.
From the values of herding parameters ($\rho_3 \sim 10\rho$) we find the surprising fact that the herding effect is much stronger for an agent in the fundamentalistic state than in the noise trader state. A possible interpretation may be the following. A typical fundamentalistic trader may be a fond manager. She follows the present majority if herding dominates. In the opposite strategy not to follow the trend a heavy risk for her career is buried. If the majority turns out to

be successful she will belong to the few who experience losses and will lose her job with no opportunity to revise her strategy. This does not happen if she has followed the majority. On the long run agents not following the general trend will die out. Therefore there exists a strong tendency towards herding behaviour also for the fundamentalists.

Acknowlegments: The author thanks Thomas Lux for stimulating discussions and the numerical data of a simulation of the LMM. He is grateful to S.Alfarano for discussions and Dr. Pierdzioch from the Istitut für Weltwirtschaft for the access to the DAX data.

References

[1] C. G. de Vries The handbook of International Macroeconomics, Blackwell, Oxford 1994, p. 348.

[2] A. Pagan Journal of Empirical Finance **3** (1996) 15

[3] E. F. Fama J. Finance **25** (1970) 383

[4] Y. Liu, P. Gopikrishnan, P. Cizeau, M. Meyer, C. K. Peng and H. E. Stanley Phys.Rev E **60** (1999) 1390 and P. Gopikrishnan, V. Plerou,L. A. Nunes Amaral, M. Meyer and H. E. Stanley Phys.Rev E **60** (1999) 5305

[5] B. Mandelbrot Journal of Business **35** (1963) 394

[6] I. N. Lobato and N. E. Savin Journal of Business and Economic Statistics **16** (1998) 261

[7] B. Mandelbrot and J. R. Wallis Water Resources Res. **4** (1968) 909

[8] B. LeBaron, Journ. Econ. Dynamics and Control 24(2000)679 and I. D. Farmer Int. J. Theor. Appl. Fin. **3** (2000) 451

[9] E. Samanidou, E. Zschischang, D. Stauffer and T. Lux in F. Schweitzer (ed.) Microscopic models for Economic Dynamics, Lecture notes in Physics, Springer, Berlin 2002.

[10] G. Kim and H.M. Markowitz Journ. Portfolio Management **16** (1989) 45

[11] P. Bak, M. Paczuski and M. Shubik Physica A **246** (1997) 430

[12] T. Lux and M. Marchesi Nature **397** (1999) 397 and Int. Journ. Theor. Appl. Finance **3** (2000) 67

[13] Y.C. Zhang Physica A **269** (1999) 30

[14] S. Solomon and R. Richmond, Physica A **299** (2001) 188 and O. Biham, Z. F. Huang, O. Malcai and S. Solomon Phys. Rev. E **64** (2001) 101

[15] A.H. Sato and H. Takayasu Phys A **250** (1998) 231

[16] G. Iori Int. J. Mod. Phys. C **10** (1999) 1149

[17] M. Levy, H. Levy and S. Solomon Journal de Physique I **5** (1995) 1087 and Physica A **242** (1997) 90

[18] G. Caldarelli,M. Marsili and Y.C. Zhang Europhys. Lett. **40** (1998) 479

[19] R. Cont and J. P. Bouchaud Macroeconomic Dynamics **4** (2000) 170 and D. Stauffer Advances in Complex Systems **4** (2001) 19

[20] D. Chowdhury and D. Stauffer Eur. Phys. Jounal B **8** (1999) 447

[21] RS. Bornholdt Int. Journal of Mod. Physics C **12** (2001) 667

[22] A. Kirman Quart. Journal Econ. **108** (1993) 137

[23] M. Aoki, Journal of Economic Behaviour and Organization (2000)

[24] S. Alfarano and T. Lux Kiel 2001

[25] Data basis of Institut für Weltwirtschaft, Kiel

[26] J. Voit The Statistical Mechanics of Financial Markets Springer, Berlin 2001

[27] D. Helbing and D. Kern Physica A **287** (2000) 259

[28] J. D. Farmer Journ. Econ. Behav. Organ. (1998), http: /xxx.lanl.gov/abs/ adaptorg/9812005

[29] M. M. Mansour and A. de Palma Physica A **128** (1984) 377

[30] E. Egenter, T. Lux and D. Stauffer Physica A **268** 1999) 268

[31] J. Honerkamp Stochastische dynamische Systeme VHC Verlagsgesellschaft Weinheim 1990